Evaluation of moments of quadratic forms in normal variables

Magnus, J.R.; Pesaran, B.

Publication date:
1990

CentER for Economic Research

Discussion paper

TILBURG UNIVERSITY

No. 9021

EVALUATION OF MOMENTS OF QUADRATIC FORMS IN NORMAL VARIABLES

by Jan R. Magnus
and Bahram Pesaran

April 1990

ISSN 0924-7815

Evaluation of moments of quadratic forms in normal variables

by

Jan R. Magnus
London School of Economics
and
CentER, Tilburg University

and

Bahram Pesaran
Economics Division, Bank of England

February 1990

Keywords: Quadratic forms; Moments; Cumulants

ABSTRACT

We present and describe a subroutine, called CUM, which calculates the cumulants and moments of a quadratic form in normal variables. That is, if we let $Q = x'Ax$, then CUM can calculate $E(Q^s)$, where x is an $n \times 1$ vector of normally distributed variables with some mean and a positive definite (hence non-singular) covariance matrix , and A is an $n \times n$ symmetric matrix. A diskette containing the Fortran code of CUM and some test programmes is available on request.

Language

Fortran 77

Description and Purpose

The subroutine CUM calculates the exact cumulants and moments of the quadratic form x'Ax, where x is an nx1 vector of normally distributed variables with some mean μ and a positive definite (hence non-singular) variance-covariance matrix Ω and A is an nxn symmetric matrix. The routine is based on theory developed by Magnus(1978, 1979) for the central case where μ=0, and Magnus(1986, Lemmas 2 and 3) for the general case.

Parameter statements

The following parameters have been set in subroutines CUM and PARINT :

ISPAR=77 & ISDIM=12	The dimensions of the array ISPRTN (ISPARxISDIM) where all possible partitions for a particular s are stored. This two dimensional array is set up by subroutine PARINT.
MAXMOM=24	The maximum of s allowed.

If 12<s≤24 is to be calculated, then both ISPAR and ISDIM should be increased.

Working out cumulants and moments for s>24 is possible. In that case not only ISPAR and ISDIM should be changed in the parameter statements, but also MAXMOM. Furthermore, the DATA statement in subroutine PARINT should be extended to include MAXMOM numbers. In the data statement the vector NUM has been set up to contain the number of all possible partitions of numbers up to 24. If a MAXMOM bigger than 24 is specified, the data statement for NUM should be extended accordingly. For a table containing the partitions for integers up to 100 see Hall(1986, p.38).

Structure

SUBROUTINE CUM(IMOM, N, LS, A, IEMU, EMU, IOMEGA, OMEGA, RKUM, RMOM, WORK1, WORK2, WORK3, VEC, IFAIL)

Formal Parameters

IMOM	integer	input:	=0 if only cumulants are required =1 if both cumulants and moments are required.
N	integer	input:	No of observations n
LS	integer	input:	order of the highest cumulant or moment required.
		output:	unchanged unless LS>M where M=MIN(MAXMOM,ISDIM) in which case LS is set equal to M.
A	real array of dimension at least Nx(N+1)/2	input:	symmetric matrix in the quadratic form x'Ax. Only the lower part of A is stored as: $a_{11}, a_{21}, a_{22}, a_{31}, a_{32}, a_{33}$ etc.
IEMU	integer	input:	=0 if μ=0 ≠0 if $\mu \neq 0$
EMU	real array of dimension at least N	input:	vector μ. Values required only if IEMU≠0 though storage should be allocated to it.
IOMEGA	integer	input:	= -1 if L^{-1} is supplied where $\Omega = LL'$, L lower triangular = 1 if L is supplied where $\Omega = LL'$, L lower triangular = 2 if Ω is supplied
OMEGA	real array of dimension at least Nx(N+1)/2	input:	either L or L^{-1} or Ω where only lower part of these are stored

RKUM	real array of dimension at least LS	output:	the required cumulants with i-th cumulant stored in RKUM(i).
RMOM	real array of dimension at least LS	output:	the required moments with i-th moment stored as RMOM(i).
WORK1	real array of dimension at least Nx(N+1)/2		work space
WORK2	real array of dimension at least Nx(N+1)/2		work space
WORK3	real array of dimension at least Nx(N+1)/2		work space
VEC	real array of dimension at least N		work space
IFAIL	integer	output:	a fault indicator where: 0: no error 1: N ≤ 1 2: IOMEGA out of range 3: LS < 1 4: if IOMEGA=2 Ω not pos. definite if IOMEGA=1 diagonal elements of L not all positive if IOMEGA=-1 diagonal elements of L^{-1} not all positive 5: if IOMEGA=2 or 1, L can't be inverted if IOMEGA=-1, L^{-1} can't be inverted 6: ISPAR in the parameter statement is too small

Auxiliary Algorithms

CUM calls various functions and subroutines listed below:

(F1)	FUNCTION INX(I,J):	picks out the appropriate element of a symmetric matrix stored in lower triangular form
(F2)	REAL*8 FUNCTION FACT(N):	calculates N! A real*8 function is used because large values of N are passed as the argument.
(S1)	SUBROUTINE PARINT:	constructs the matrix containing all partitions of an integer
(S2)	SUBROUTINE SEP:	decomposes A (pos. def.) into LL' (L lower triangular) and replaces A by L
(S3)	SUBROUTINE LOWINV:	inversion (in place) of a lower triangular matrix
(S4)	SUBROUTINE MULT1:	computes C=B'AB (A symmetric, B lower triangular)

Constants

The DATA statement in CUM sets EPS = 1.0-11 as a small number. Any number with an absolute value below EPS will be treated as zero.

Precision

The version of the routines listed below is in double precision (Real*8). In order to change the program to single precision the following changes should be made:

(1) Change all IMPLICIT REAL*8 to IMPLICIT REAL*4

(2) Change the constants in the DATA statements to single precision versions.

(3) Change DSQRT, DABS to SQRT, ABS, in the statement functions appearing in the beginning of routines SEP and LOWINV.

Time

Using the VAX 6330 at the London School of Economics, typical CPU times for the double precision version are reported in Table 1 for different combinations of LS(4, 8 and 12) and n(10, 20, 30, 40 and 50). Other parameters were kept fixed at IMOM=1 and IEMU=1 as the variations in these did not affect the CPU time significantly. It should be pointed out that the effect of increase in LS is cumulative, i.e. when LS=4 all of the first 4 cumulants and moments are calculated while for LS=8 all first 8 cumulants and moments are evaluated by CUM.

Table 1. **Typical CPU times**

n	LS=4	LS=8	LS=12
10	0.30	0.32	0.47
20	1.07	1.47	1.98
30	3.74	5.17	6.39
40	10.43	13.24	16.35
50	23.81	29.52	35.26

Accuracy

In order to check the accuracy of calculations we used CUM to evaluate the first 4 cumulants and moments of the χ^2 distribution with 4 degrees of freedom. This was achieved by using an arbitrary (20x4) matrix R and by setting $A=R(R'R)^{-1}R'$ and $\Omega=I$. Setting IEMU=0 the first 4 cumulants and moments of $x'Ax$ were calculated.

The exact rth cumulant of a χ^2 distribution with ν degrees of freedom can be calculated using the formula

$$(1) \qquad \kappa_r = \nu \, 2^{r-1} \, (r-1)!$$

while the moments of a χ^2 distribution with ν degrees of freedom can be worked out using its moment generating function which is

$$(2) \qquad MGM(t) = (1 - 2t)^{-\nu/2}$$

Table 2 allows the comparison between the exact results using (1) and (2) and the calculated values using CUM.

Table 2. Accuracy of cumulants and moments of a χ^2 distribution calculated by CUM

	Cumulants		Moments	
r	Exact	Calculated	Exact	Calculated
1	4	4.00000000	4	4.0000000
2	8	8.00000000	24	24.00000000
3	32	32.00000000	192	192.00000000
4	192	192.00000000	1920	1920.00000000

Examining Table 2 shows that in this example an accuracy of at least eight decimal points is achieved.

Related Algorithms

The algorithm for the evaluation of moments of ratios of quadratic forms in normal variables and related statistics by Magnus and Pesaran (1990).

REFERENCES

Hall, M.(1986), Combinatorial Theory(2nd edition), John Wiley and Sons, New York.

Magnus, J. R.(1978), 'The moments of products of quadratic forms in normal variables', Statistica Neerlandica, 32, 201-210.

Magnus, J. R.(1979), 'The expectation of products of quadratic forms in normal variables: the practice', Statistica Neerlandica, 33, 131-136.

Magnus, J. R.(1986), 'The exact moments of a ratio of quadratic forms in normal variables', Annales d'Economie et de Statistique, 4, 95-109.

Magnus, J. R. and Pesaran, B.(1990). 'Evaluation of moments of ratios of quadratic forms in normal variables and related statistics', submitted to Applied Statistics.

```
      SUBROUTINE CUM(IMOM,N,LS,A,IEMU,EMU,IOMEGA,
     +OMEGA,RKUM,RMOM,WORK1,WORK2,WORK3,VEC,IFAIL)
C
C        ALGORITHM AS??? APPL. STATIST. (1990).
C        CALCULATES CUMULANTS AND MOMENTS OF A
C        QUADRATIC FORM IN NORMAL VARIABLES
C
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER(ISDIM=12,ISPAR=77,MAXMOM=24)
      DIMENSION A(*),OMEGA(*),WORK1(*),WORK2(*),WORK3(*)
      DIMENSION EMU(*),RKUM(*),RMOM(*),VEC(*)
      DIMENSION ISPRTN(ISPAR,ISDIM),TR(ISDIM)
      DATA ZERO,ONE,TWO,EPS/0.0D0,1.0D0,2.0D0,1.D-11/
      IFAIL=1
      IF(N.LE.1) RETURN
      IFAIL=2
      IF(IOMEGA.NE.-1.AND.IOMEGA.NE.1.AND.IOMEGA.NE.2) RETURN
      IFAIL=3
      IF(LS.LT.1) RETURN
      NN=N*(N+1)/2
      LS=MIN(LS,MAXMOM,ISDIM)
C
C        SET WORK2 = L WHERE OMEGA = LL'
C
      DO 10 I=1,NN
10    WORK2(I)=OMEGA(I)
      IF(IOMEGA.EQ.1.OR.IOMEGA.EQ.-1) THEN
        IFAIL=4
        DO 20 I=1,N
        R=WORK2(INX(I,I))
        IF(R.LT.EPS) RETURN
20      CONTINUE
      END IF
```

```
      IFAIL=0
      IF(IOMEGA.EQ.2) CALL SEP(WORK2,N,ONE,EPS,IFAIL)
      IF(IOMEGA.EQ.-1) CALL LOWINV(WORK2,N,ZERO,ONE,EPS,IFAIL)
      IF(IFAIL.NE.0) THEN
        IF(IOMEGA.EQ.2) IFAIL=4
        IF(IOMEGA.EQ.-1) IFAIL=5
        RETURN
      END IF
C
C       SET UP L'AL IN WORK1
C
      CALL MULT1(A,WORK2,WORK1,N,ZERO)
C
C       CALCULATE LINV*EMU IN VEC IF EMU IS NOT ZERO
C
      IF(IEMU.NE.0) THEN
        IF(IOMEGA.GT.0) THEN
          CALL LOWINV(WORK2,N,ZERO,ONE,EPS,IFAIL)
          IFAIL=IFAIL+4
          IF(IFAIL.NE.4) RETURN
        END IF
        DO 40 I=1,N
        SUM=ZERO
        DO 30 J=1,I
        IF(IOMEGA.GT.0) THEN
          R=WORK2(INX(I,J))
        ELSE
          R=OMEGA(INX(I,J))
        END IF
30      SUM=SUM+R*EMU(J)
40      VEC(I)=SUM
      END IF
```

```
C
C       CALCULATE CUMULANTS OF QUADRATIC FORM IN RKUM
C
      DO 130 IS=1,LS
      IF(IS.EQ.1) THEN
        DO 50 I=1,NN
50      WORK2(I)=WORK1(I)
      ELSE
        DO 70 I=1,N
        DO 70 J=1,I
        IJ=INX(I,J)
        SUM=ZERO
        DO 60 K=1,N
        IK=INX(I,K)
        KJ=INX(K,J)
60      SUM=SUM+WORK1(IK)*WORK2(KJ)
70      WORK3(IJ)=SUM
        DO 80 I=1,NN
80      WORK2(I)=WORK3(I)
      END IF
      SUM=ZERO
      DO 90 I=1,N
      RI=WORK2(INX(I,I))
90    SUM=SUM+RI
      TR(IS)=SUM
      IF(IEMU.NE.0) THEN
        SUM=ZERO
        DO 100 I=1,N
        TT=TWO
        DO 100 J=1,I
        IJ=INX(I,J)
        IF(I.EQ.J) TT=ONE
100     SUM=SUM+TT*VEC(I)*WORK2(INX(I,J))*VEC(J)
        RS=IS
```

```
        TR(IS)=TR(IS)+RS*SUM
      END IF
      TT=2**(IS-1)
      RKUM(IS)=TT*FACT(IS-1)*TR(IS)
C
C       CALCULATE MOMENTS OF QUADRATIC FORM (IF REQUIRED) IN RMOM
C
      IF(IMOM.EQ.1) THEN
        CALL PARINT(IS,ISPRTN,ISPAR,ISDIM,ISROW,IFAIL)
        IF(IFAIL.NE.0) THEN
          IFAIL=6
          RETURN
        END IF
        SUM=ZERO
        DO 120 I=1,ISROW
        RB=ONE
        RA=ONE
        DO 110 L=1,IS
        R2L=2*L
        K=ISPRTN(I,L)
        IF(K.NE.0) THEN
          RB=RB*FACT(K)*(R2L**K)
          RA=RA*(TR(L)**K)
        END IF
110     CONTINUE
        RB=(FACT(IS)*(TWO**IS))/RB
120     SUM=SUM+RA*RB
        RMOM(IS)=SUM
      END IF
130   CONTINUE
      IFAIL=0
      RETURN
      END
```

```
      SUBROUTINE PARINT(M,MPRTN,MRDIM,MDIM,MR,IFAIL)
C
C       CONSTRUCTS THE MR X M MATRIX "MPRTN" CONTAINING
C       ALL MR PARTITIONS OF THE INTEGER M
C
      PARAMETER(MAXMOM=24)
      DIMENSION MPRTN(MRDIM,MDIM)
      DIMENSION NUM(MAXMOM),IWORK(MAXMOM)
      DATA NUM/1,2,3,5,7,11,15,22,30,42,56,77,101,135,176,231,
     +297,385,490,627,792,1002,1255,1575/
      IFAIL=1
      IF(M.LT.1.OR.M.GT.MAXMOM.OR.M.GT.MDIM) RETURN
      IFAIL=2
      IF(NUM(M).GT.MRDIM) RETURN
      IFAIL=0
      N1=0
      N2=1
      N3=0
      MR=1
      M1=1
      L=0
      MPRTN(1,1)=1
      IF(M.EQ.1) RETURN
      DO 10 J=2,M
10    MPRTN(1,J)=0
      DO 99 K=2,M
      IF(N2.NE.0) THEN
        DO 30 I=1,N2
        MPRTN(MR+I,1)=0
        DO 20 J=2,M
20      MPRTN(MR+I,J)=MPRTN(I+N1,J)
30      MPRTN(MR+I,2)=MPRTN(MR+I,2)+1
      END IF
      IF(N3.NE.0) THEN
```

```
      L=0
      DO 80 I=N1+N2+1,MR
      DO 80 J=2,K-1
      IF(MPRTN(I,J).EQ.0) GO TO 80
      DO 40 JJ=1,M
40    IWORK(JJ)=MPRTN(I,JJ)
      IWORK(J)=IWORK(J)-1
      IWORK(J+1)=IWORK(J+1)+1
      IF(N2.NE.0.OR.L.NE.0) THEN
        DO 60 II=MR+1,MR+N2+L
        DO 50 JJ=1,M-1
50      IF(MPRTN(II,JJ).NE.IWORK(JJ)) GO TO 60
        IF(MPRTN(II,M).EQ.IWORK(M)) GO TO 80
60      CONTINUE
      END IF
      L=L+1
      DO 70 JJ=1,M
70    MPRTN(MR+N2+L,JJ)=IWORK(JJ)
80    CONTINUE
    END IF
    DO 90 II=1,MR
90  MPRTN(II,1)=MPRTN(II,1)+1
    N1=M1
    N3=N2+L
    N2=MR-N1
    M1=MR
    MR=MR+N3
99  CONTINUE
    RETURN
    END
```

```
      INTEGER FUNCTION INX(I,J)
C
C       PICKS OUT THE APPROPRIATE ELEMENT OF A SYMMETRIC
C       MATRIX STORED IN LOWER TRIANGULAR FORM
C
      IF(I.GE.J) THEN
        INX=I*(I-1)/2+J
      ELSE
        INX=J*(J-1)/2+I
      END IF
      RETURN
      END

      REAL*8 FUNCTION FACT(N)
C
C       CALCULATES N FACTORIAL
C
      FACT=1
      IF(N.LE.1) RETURN
      DO 10 I=2,N
      RI=I
10    FACT=FACT*RI
      RETURN
      END

      SUBROUTINE SEP(A,N,ONE,EPS,IFAIL)
C
C       DECOMPOSES A (POSITIVE DEFINITE) INTO LL'
C       (L LOWER TRIANGULAR) AND REPLACES A BY L
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(*)
      ZSQRT(H)=DSQRT(H)
      IFAIL=1
```

```
      DO 20 I=1,N
      DO 20 J=I,N
      IJ=INX(J,I)
      R=A(IJ)
      IF(I.NE.1) THEN
        DO 10 K=1,I-1
10      R=R-A(INX(I,K))*A(INX(J,K))
      END IF
      IF(I.EQ.J) THEN
        IF(R.LT.EPS) RETURN
        D=ONE/ZSQRT(R)
      END IF
20    A(IJ)=R*D
      IFAIL=0
      RETURN
      END

      SUBROUTINE LOWINV(A,N,ZERO,ONE,EPS,IFAIL)
C
C       INVERSION (IN PLACE) OF N X N LOWER TRIANGULAR MATRIX
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(*)
      ZABS(H)=DABS(H)
      IFAIL=1
      DO 20 I=1,N
      R=A(INX(I,I))
      IF(ZABS(R).LT.EPS) RETURN
      D=ONE/R
      DO 20 J=1,I
      IJ=INX(I,J)
      IF(I.NE.J) THEN
        SUM=ZERO
        DO 10 K=J,I-1
```

```
10        SUM=SUM+A(INX(I,K))*A(INX(K,J))
          A(IJ)=-SUM*D
        ELSE
          A(IJ)=D
        END IF
20      CONTINUE
        IFAIL=0
        RETURN
        END

        SUBROUTINE MULT1(A,B,C,N,ZERO)
C
C         FINDS C = B'AB (A=A', B LOWER TRIANGULAR)
C         ALL MATRICES N X N
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION A(*),B(*),C(*)
        DO 20 I=1,N
        DO 20 J=1,I
        IJ=INX(I,J)
        SUM=ZERO
        DO 10 K=I,N
        KI=INX(K,I)
        DO 10 L=J,N
        LJ=INX(L,J)
        KL=INX(K,L)
10      SUM=SUM+B(KI)*A(KL)*B(LJ)
20      C(IJ)=SUM
        RETURN
        END
```

Discussion Paper Series, CentER, Tilburg University, The Netherlands:

(For previous papers please consult previous discussion papers.)

No.	Author(s)	Title
8901	Th. Ten Raa and P. Kop Jansen	The Choice of Model in the Construction of Input-Output Coefficients Matrices
8902	Th. Nijman and F. Palm	Generalized Least Squares Estimation of Linear Models Containing Rational Future Expectations
8903	A. van Soest, I. Woittiez, A. Kapteyn	Labour Supply, Income Taxes and Hours Restrictions in The Netherlands
8904	F. van der Ploeg	Capital Accumulation, Inflation and Long-Run Conflict in International Objectives
8905	Th. van de Klundert and A. van Schaik	Unemployment Persistence and Loss of Productive Capacity: A Keynesian Approach
8906	A.J. Markink and F. van der Ploeg	Dynamic Policy Simulation of Linear Models with Rational Expectations of Future Events: A Computer Package
8907	J. Osiewalski	Posterior Densities for Nonlinear Regression with Equicorrelated Errors
8908	M.F.J. Steel	A Bayesian Analysis of Simultaneous Equation Models by Combining Recursive Analytical and Numerical Approaches
8909	F. van der Ploeg	Two Essays on Political Economy (i) The Political Economy of Overvaluation (ii) Election Outcomes and the Stockmarket
8910	R. Gradus and A. de Zeeuw	Corporate Tax Rate Policy and Public and Private Employment
8911	A.P. Barten	Allais Characterisation of Preference Structures and the Structure of Demand
8912	K. Kamiya and A.J.J. Talman	Simplicial Algorithm to Find Zero Points of a Function with Special Structure on a Simplotope
8913	G. van der Laan and A.J.J. Talman	Price Rigidities and Rationing
8914	J. Osiewalski and M.F.J. Steel	A Bayesian Analysis of Exogeneity in Models Pooling Time-Series and Cross-Section Data
8915	R.P. Gilles, P.H. Ruys and J. Shou	On the Existence of Networks in Relational Models

No.	Author(s)	Title
8916	A. Kapteyn, P. Kooreman and A. van Soest	Quantity Rationing and Concavity in a Flexible Household Labor Supply Model
8917	F. Canova	Seasonalities in Foreign Exchange Markets
8918	F. van der Ploeg	Monetary Disinflation, Fiscal Expansion and the Current Account in an Interdependent World
8919	W. Bossert and F. Stehling	On the Uniqueness of Cardinally Interpreted Utility Functions
8920	F. van der Ploeg	Monetary Interdependence under Alternative Exchange-Rate Regimes
8921	D. Canning	Bottlenecks and Persistent Unemployment: Why Do Booms End?
8922	C. Fershtman and A. Fishman	Price Cycles and Booms: Dynamic Search Equilibrium
8923	M.B. Canzoneri and C.A. Rogers	Is the European Community an Optimal Currency Area? Optimal Tax Smoothing versus the Cost of Multiple Currencies
8924	F. Groot, C. Withagen and A. de Zeeuw	Theory of Natural Exhaustible Resources: The Cartel-Versus-Fringe Model Reconsidered
8925	O.P. Attanasio and G. Weber	Consumption, Productivity Growth and the Interest Rate
8926	N. Rankin	Monetary and Fiscal Policy in a 'Hartian' Model of Imperfect Competition
8927	Th. van de Klundert	Reducing External Debt in a World with Imperfect Asset and Imperfect Commodity Substitution
8928	C. Dang	The D_1-Triangulation of R^n for Simplicial Algorithms for Computing Solutions of Nonlinear Equations
8929	M.F.J. Steel and J.F. Richard	Bayesian Multivariate Exogeneity Analysis: An Application to a UK Money Demand Equation
8930	F. van der Ploeg	Fiscal Aspects of Monetary Integration in Europe
8931	H.A. Keuzenkamp	The Prehistory of Rational Expectations

No.	Author(s)	Title
8932	E. van Damme, R. Selten and E. Winter	Alternating Bid Bargaining with a Smallest Money Unit
8933	H. Carlsson and E. van Damme	Global Payoff Uncertainty and Risk Dominance
8934	H. Huizinga	National Tax Policies towards Product-Innovating Multinational Enterprises
8935	C. Dang and D. Talman	A New Triangulation of the Unit Simplex for Computing Economic Equilibria
8936	Th. Nijman and M. Verbeek	The Nonresponse Bias in the Analysis of the Determinants of Total Annual Expenditures of Households Based on Panel Data
8937	A.P. Barten	The Estimation of Mixed Demand Systems
8938	G. Marini	Monetary Shocks and the Nominal Interest Rate
8939	W. Güth and E. van Damme	Equilibrium Selection in the Spence Signaling Game
8940	G. Marini and P. Scaramozzino	Monopolistic Competition, Expected Inflation and Contract Length
8941	J.K. Dagsvik	The Generalized Extreme Value Random Utility Model for Continuous Choice
8942	M.F.J. Steel	Weak Exogenity in Misspecified Sequential Models
8943	A. Roell	Dual Capacity Trading and the Quality of the Market
8944	C. Hsiao	Identification and Estimation of Dichotomous Latent Variables Models Using Panel Data
8945	R.P. Gilles	Equilibrium in a Pure Exchange Economy with an Arbitrary Communication Structure
8946	W.B. MacLeod and J.M. Malcomson	Efficient Specific Investments, Incomplete Contracts, and the Role of Market Alternatives
8947	A. van Soest and A. Kapteyn	The Impact of Minimum Wage Regulations on Employment and the Wage Rate Distribution
8948	P. Kooreman and B. Melenberg	Maximum Score Estimation in the Ordered Response Model

No.	Author(s)	Title
8949	C. Dang	The D_3-Triangulation for Simplicial Deformation Algorithms for Computing Solutions of Nonlinear Equations
8950	M. Cripps	Dealer Behaviour and Price Volatility in Asset Markets
8951	T. Wansbeek and A. Kapteyn	Simple Estimators for Dynamic Panel Data Models with Errors in Variables
8952	Y. Dai, G. van der Laan, D. Talman and Y. Yamamoto	A Simplicial Algorithm for the Nonlinear Stationary Point Problem on an Unbounded Polyhedron
8953	F. van der Ploeg	Risk Aversion, Intertemporal Substitution and Consumption: The CARA-LQ Problem
8954	A. Kapteyn, S. van de Geer, H. van de Stadt and T. Wansbeek	Interdependent Preferences: An Econometric Analysis
8955	L. Zou	Ownership Structure and Efficiency: An Incentive Mechanism Approach
8956	P.Kooreman and A. Kapteyn	On the Empirical Implementation of Some Game Theoretic Models of Household Labor Supply
8957	E. van Damme	Signaling and Forward Induction in a Market Entry Context
9001	A. van Soest, P. Kooreman and A. Kapteyn	Coherency and Regularity of Demand Systems with Equality and Inequality Constraints
9002	J.R. Magnus and B. Pesaran	Forecasting, Misspecification and Unit Roots: The Case of AR(1) Versus ARMA(1,1)
9003	J. Driffill and C. Schultz	Wage Setting and Stabilization Policy in a Game with Renegotiation
9004	M. McAleer, M.H. Pesaran and A. Bera	Alternative Approaches to Testing Non-Nested Models with Autocorrelated Disturbances: An Application to Models of U.S. Unemployment
9005	Th. ten Raa and M.F.J. Steel	A Stochastic Analysis of an Input-Output Model: Comment
9006	M. McAleer and C.R. McKenzie	Keynesian and New Classical Models of Unemployment Revisited

No.	Author(s)	Title
9007	J. Osiewalski and M.F.J. Steel	Semi-Conjugate Prior Densities in Multivariate t Regression Models
9008	G.W. Imbens	Duration Models with Time-Varying Coefficients
9009	G.W. Imbens	An Efficient Method of Moments Estimator for Discrete Choice Models with Choice-Based Sampling
9010	P. Deschamps	Expectations and Intertemporal Separability in an Empirical Model of Consumption and Investment under Uncertainty
9011	W. Güth and E. van Damme	Gorby Games - A Game Theoretic Analysis of Disarmament Campaigns and the Defense Efficiency-Hypothesis
9012	A. Horsley and A. Wrobel	The Existence of an Equilibrium Density for Marginal Cost Prices, and the Solution to the Shifting-Peak Problem
9013	A. Horsley and A. Wrobel	The Closedness of the Free-Disposal Hull of a Production Set
9014	A. Horsley and A. Wrobel	The Continuity of the Equilibrium Price Density: The Case of Symmetric Joint Costs, and a Solution to the Shifting-Pattern Problem
9015	A. van den Elzen, G. van der Laan and D. Talman	An Adjustment Process for an Exchange Economy with Linear Production Technologies
9016	P. Deschamps	On Fractional Demand Systems and Budget Share Positivity
9017	B.J. Christensen and N.M. Kiefer	The Exact Likelihood Function for an Empirical Job Search Model
9018	M. Verbeek and Th. Nijman	Testing for Selectivity Bias in Panel Data Models
9019	J.R. Magnus and B. Pesaran	Evaluation of Moments of Ratios of Quadratic Forms in Normal Variables and Related Statistics
9020	A. Robson	Status, the Distribution of Wealth, Social and Private Attitudes to Risk
9021	J.R. Magnus and B. Pesaran	Evaluation of Moments of Quadratic Forms in Normal Variables

PO. BOX 90153, 5000 LE TILBURG, THE NETHERLAND